

# Navigation Channel Infilling by Cross-Channel Transport – Screening Tool –



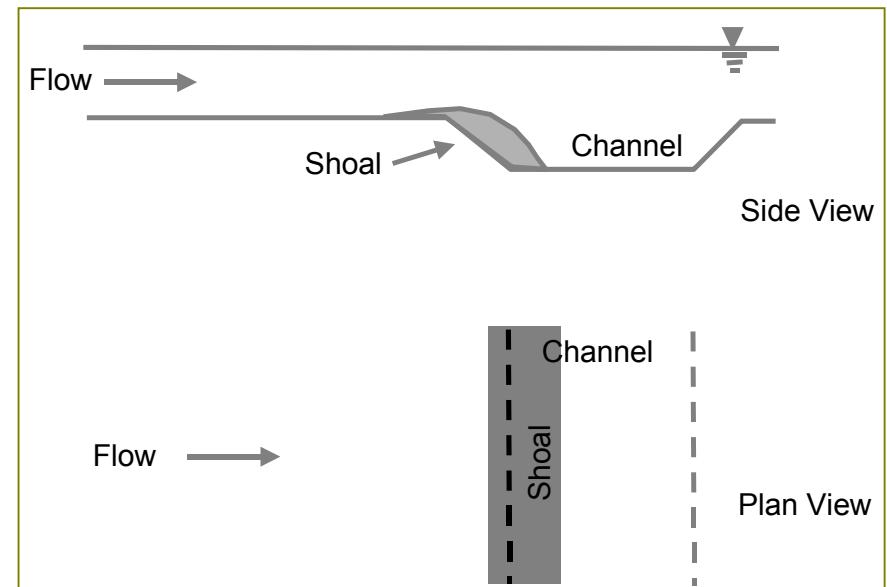
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**Coastal and Hydraulics Laboratory**

# Overview of Talk

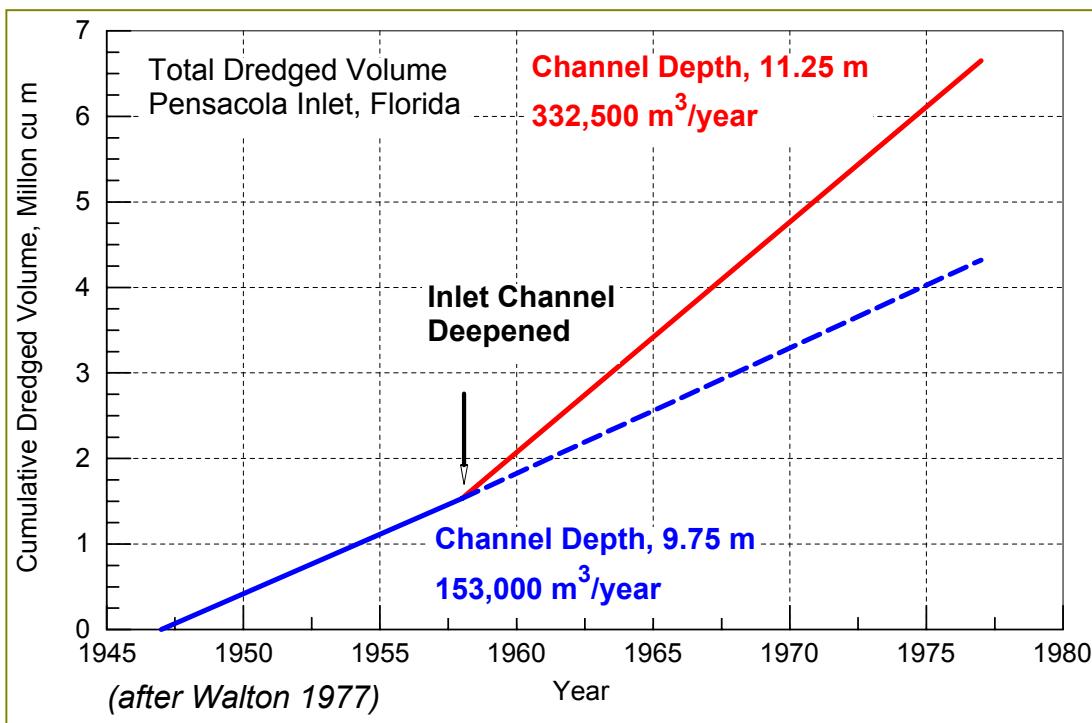
## Navigation Channel Infilling by Cross-Channel Transport – Screening Tool –



- Motivation
- Analytical Model / Screening Tool
  - Assumptions & governing equations
  - Analytical & numerical solutions
  - Sensitivity tests
  - Preliminary validation
- Trapping Ratio
- Concentration Profile
- Summary



# Channel Performance & Management



*If deepen/widen a channel, dredging is expected to increase because...*

- Deeper channel is better trap
- Wider channel is better trap
- Side slopes adjust (slump)
- Channel is longer

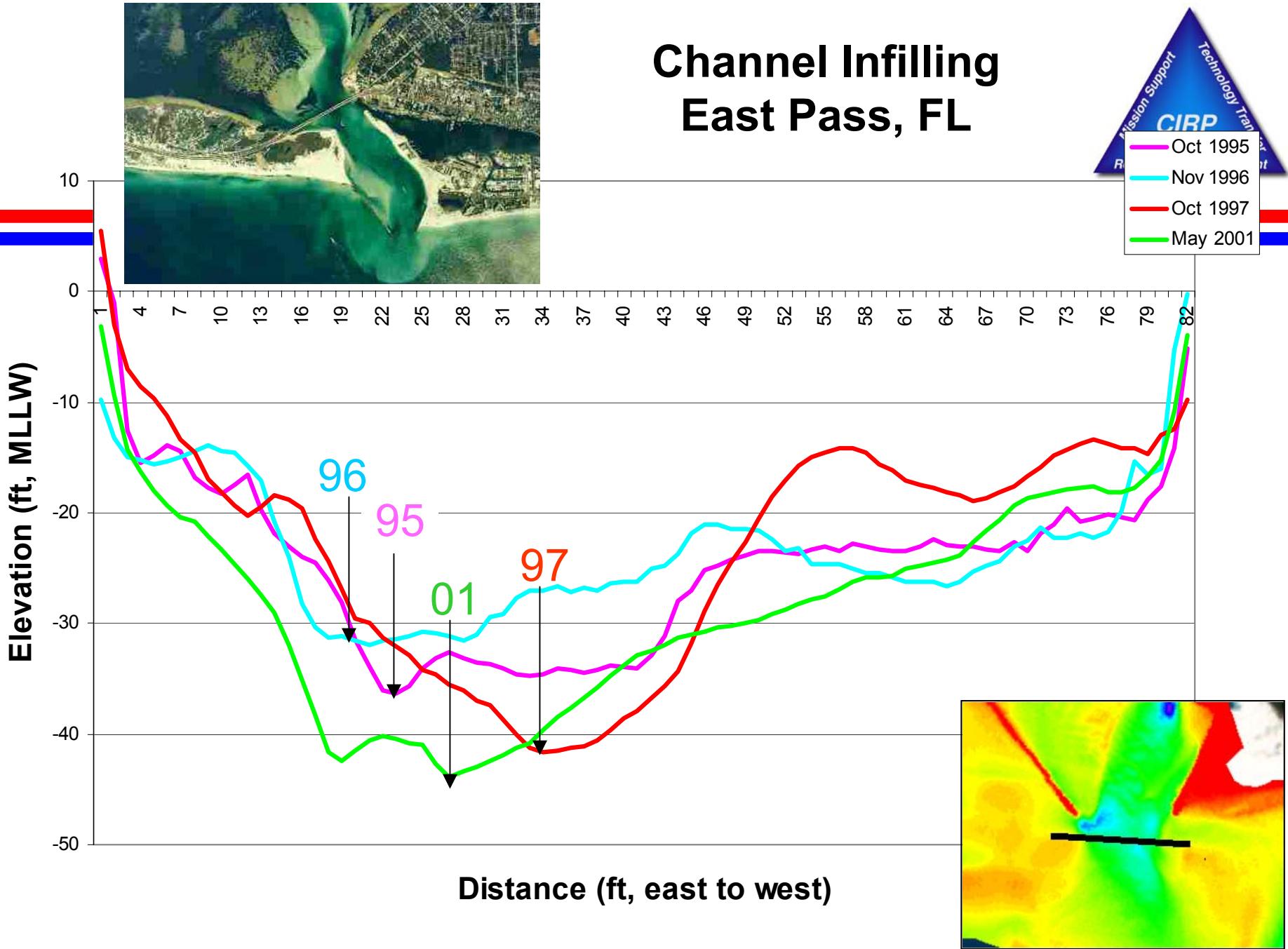


# Motivation – goals

Navigation Channel Infilling  
by Cross-Channel Transport  
– Screening Tool –

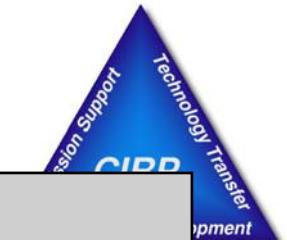
- Screening tool – assess channel modifications & performance
  - Readily available inputs (macro-processes)
  - Rapid answers
  - Robust (won't give extreme answers)
  - “What-if” easily done
- Analytical solution – reveal roles of acting processes
- Pedagogical tool
- Possibility of generalization

# Channel Infilling East Pass, FL

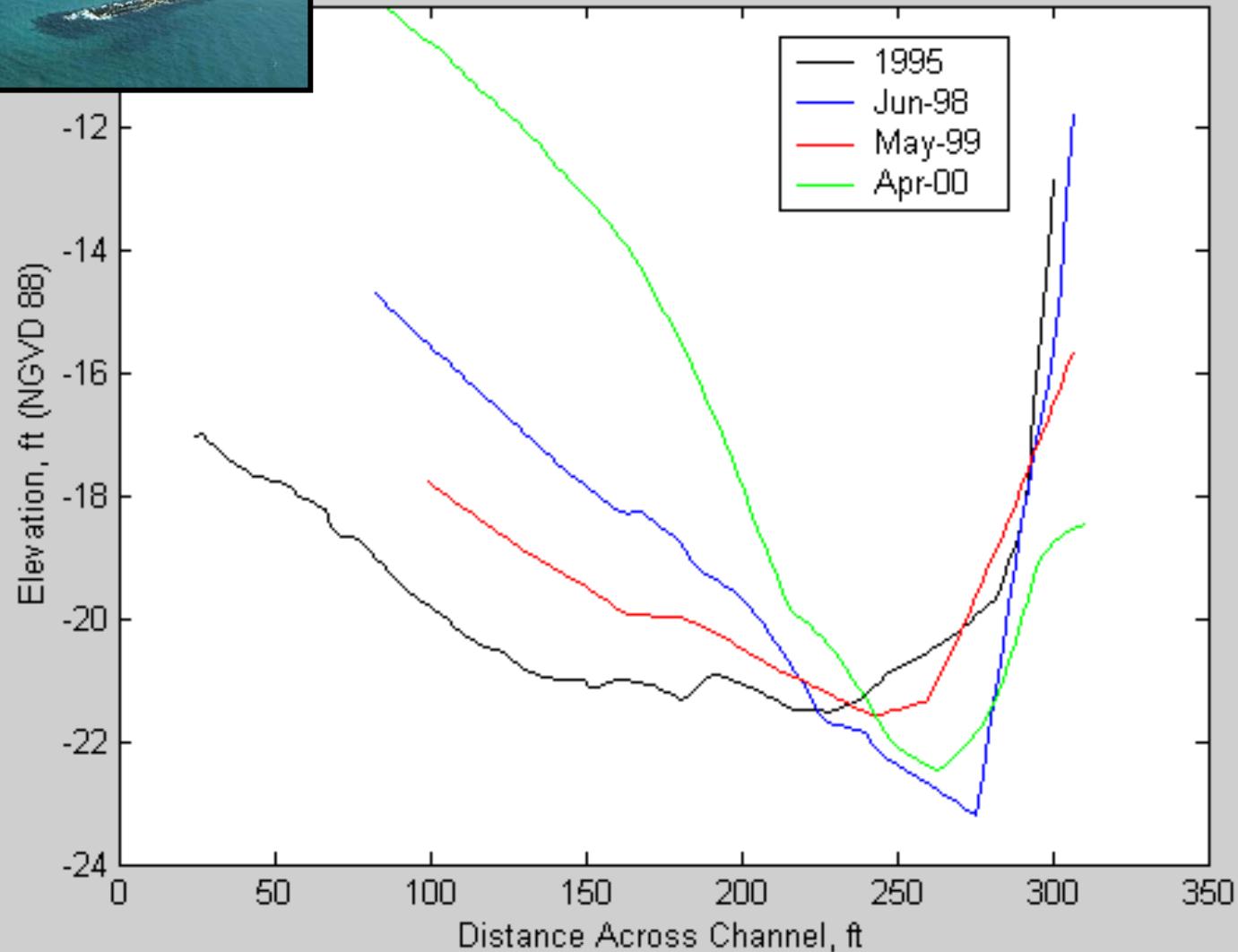




# Channel Infilling, Shark River, NJ

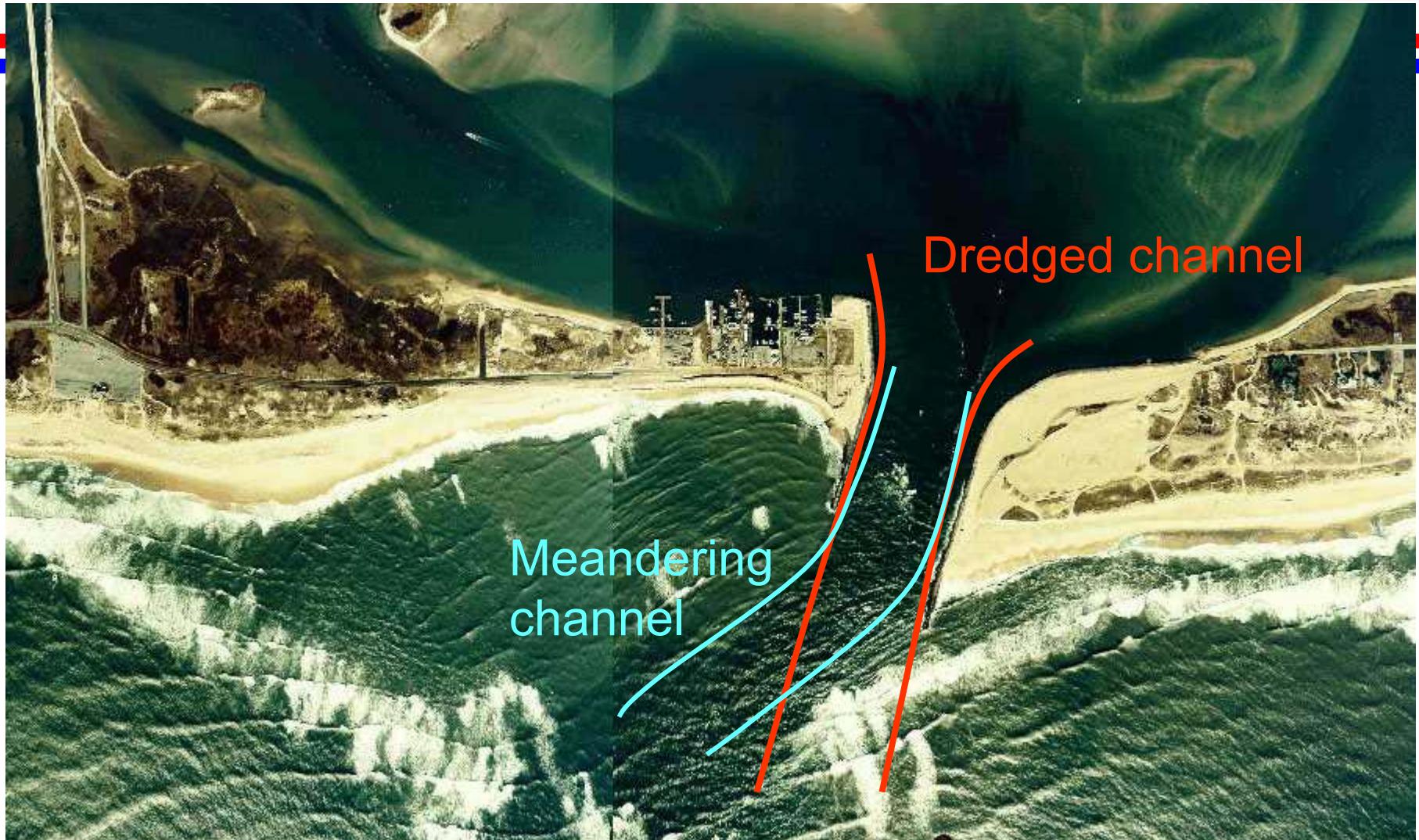


Cross-Section

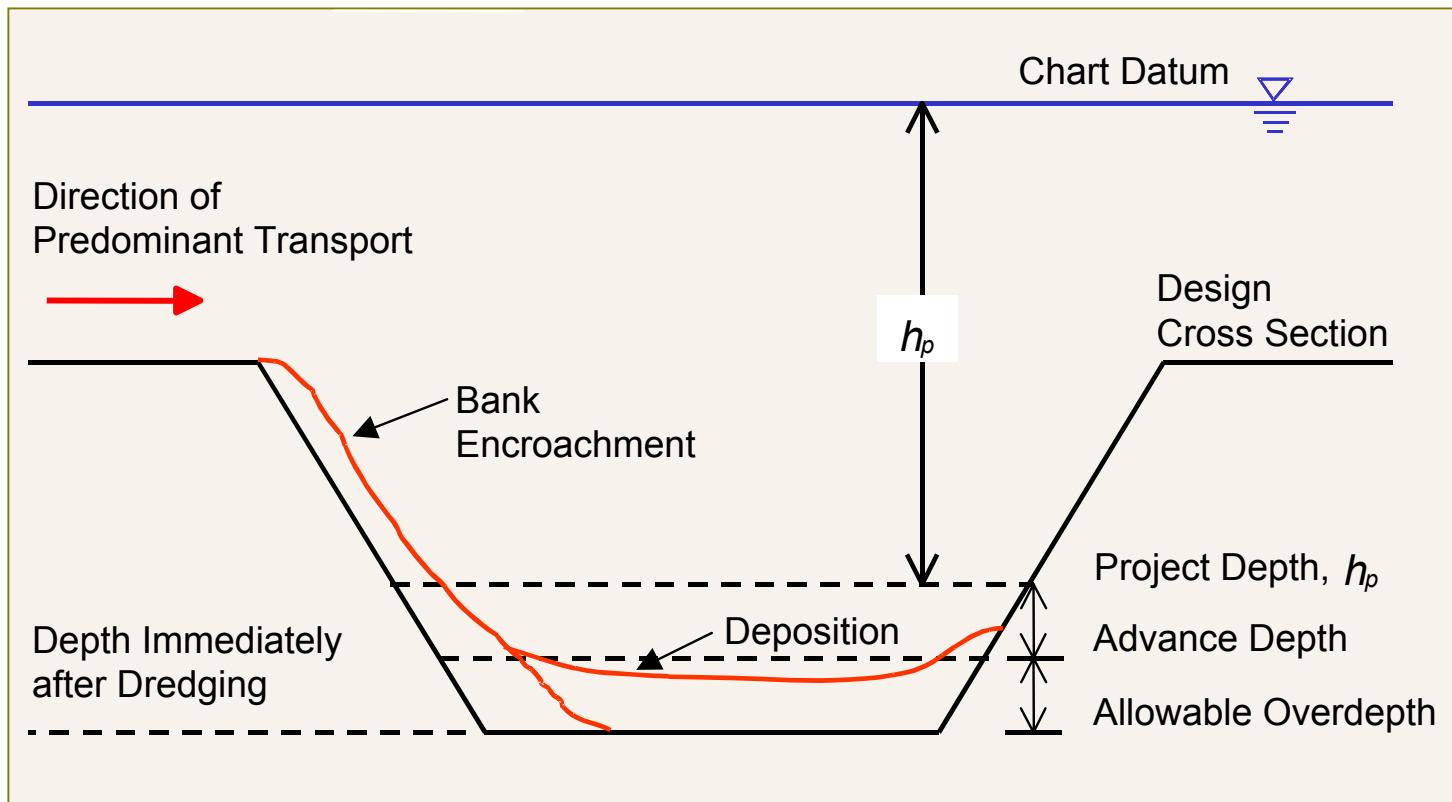


# Channel Migration

## Shinnecock Inlet, NY



# Derivation of Channel Infilling Model assumptions



# Governing Equations

## Partition of Transport

$$q = q_b + q_d + q_s \\ = (a_b + a_d + a_s)q$$

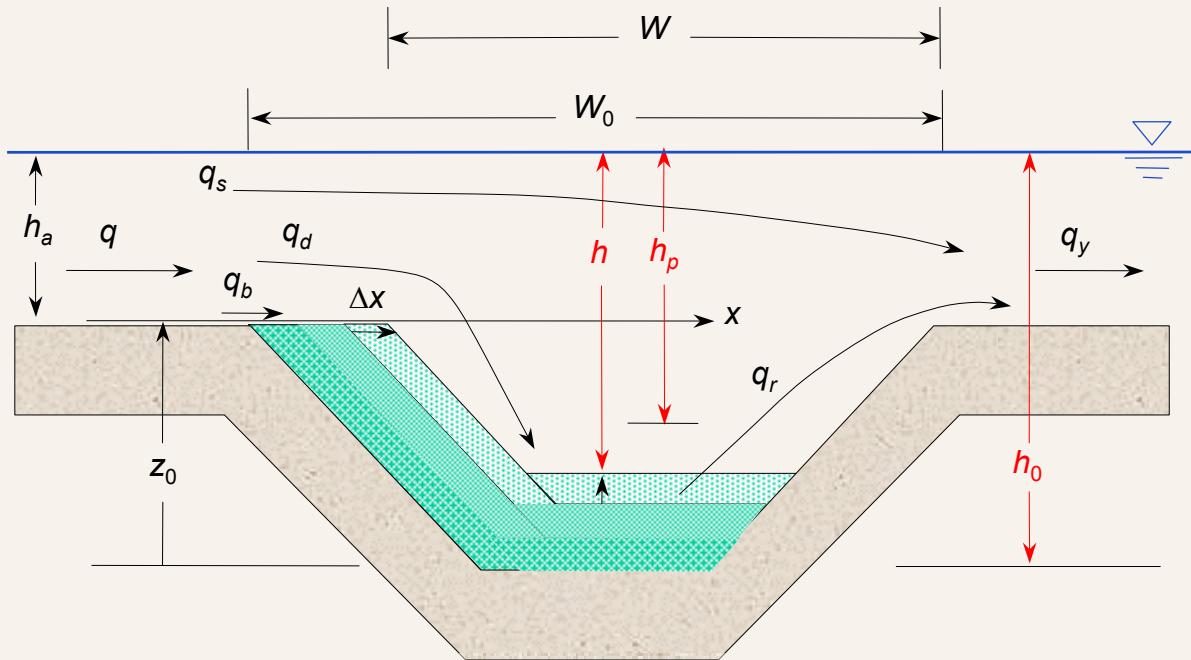
## Channel Bottom

$$\Delta z W = (q_{in} - q_{out}) \Delta t$$

$$q_{out} = \frac{z}{z_0} q_{in}$$

$$q_{in} = a_d q$$

$$\frac{dz}{dt} = \frac{a_d}{W_0 - x} \left( 1 - \frac{z}{z_0} \right) q$$



$$W(x, t) = W_0 - x(t), \quad \text{for } x < W_0$$

## Channel Side

$$\Delta x(z_0 - z) = q_b \Delta t = a_b q \Delta t$$

$$\frac{dx}{dt} = \frac{a_b}{z_0 - z} q$$

# Governing Equations

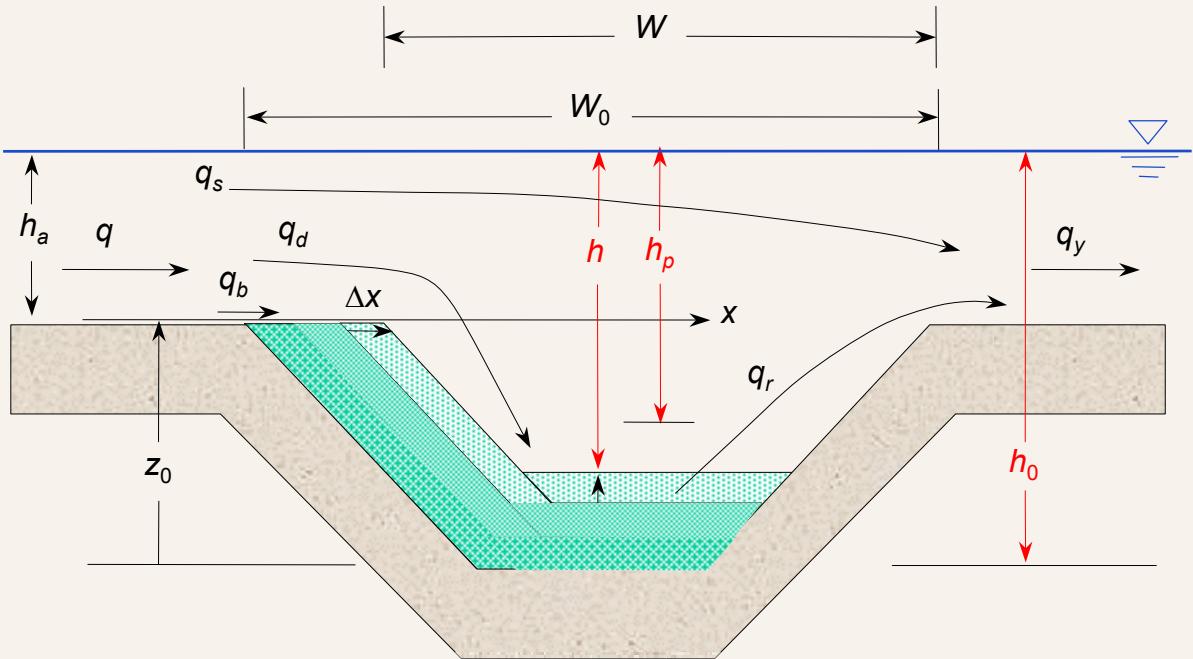
## Partition of Transport

$$q = q_b + q_d + q_s$$

$$q_c = q_b + q_d - q_r$$

$$q_y = q_s + q_r$$

$$q = q_c + q_y (= q_b + q_d + q_s)$$



$$\left. \begin{aligned} q_b &= a_b q \\ q_d &= a_d q \\ q_s &= a_s q \end{aligned} \right\} \quad a_b + a_d + a_s = 1$$

$$\left. \begin{aligned} \frac{dz}{dt} &= \frac{a_d}{W_0 - x} \left( 1 - \frac{z}{z_0} \right) q \\ \frac{dx}{dt} &= \frac{a_b}{z_0 - z} q \end{aligned} \right\} \rightarrow \begin{cases} \text{linearize} \\ z \ll z_0 \\ x \ll W_0 \end{cases}$$

$$\boxed{\begin{aligned} \frac{dz}{dt} &= \frac{a_d}{W_0} q \left( 1 - \frac{z}{z_0} + \frac{x}{W_0} \right), \quad z(0) = 0 \\ \frac{dx}{dt} &= \frac{a_b}{z_0} q \left( 1 + \frac{z}{z_0} \right), \quad x(0) = 0 \end{aligned}}$$



# Analytical Solution (linearized eqn's)

$$\frac{dz}{dt} = \frac{a_d}{W_0} q \left( 1 - \frac{z}{z_0} + \frac{x}{W_0} \right), \quad z(0) = 0$$

$$\frac{dx}{dt} = \frac{a_b}{z_0} q \left( 1 + \frac{z}{z_0} \right), \quad x(0) = 0$$



$$z = C_1 \exp(r_1 t) + C_2 \exp(r_2 t) - z_0$$

$$x = \frac{a_b}{z_0^2} q \left[ \frac{C_1}{r_1} (\exp(r_1 t) - 1) + \frac{C_2}{r_2} (\exp(r_2 t) - 1) \right]$$

Shortly after dredging ( $t$  small)

$$z = \frac{a_d}{W_0} qt - \frac{a_d}{2W_0^2 z_0} (a_d + a_b)(qt)^2$$

$$x = \frac{a_b}{z_0} qt + \frac{a_b a_d}{2W_0 z_0^2} (qt)^2$$

Infilling rate

$$R_z = \frac{dz}{dt} = \frac{a_d}{W_0} q - \frac{a_d}{W_0^2 z_0} (a_d + a_b) q^2 t$$

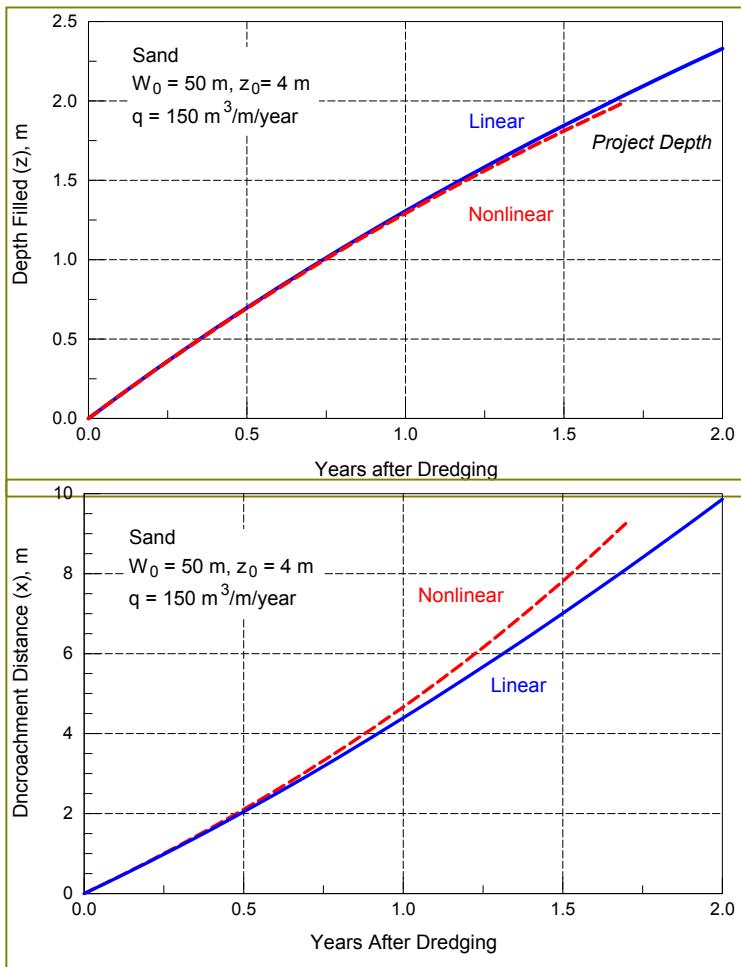
Time interval for dredging maint.

$$\Delta t_p \cong \frac{W_0 z_0}{a_d q} (h_0 - h_p)$$

Bypassing rate

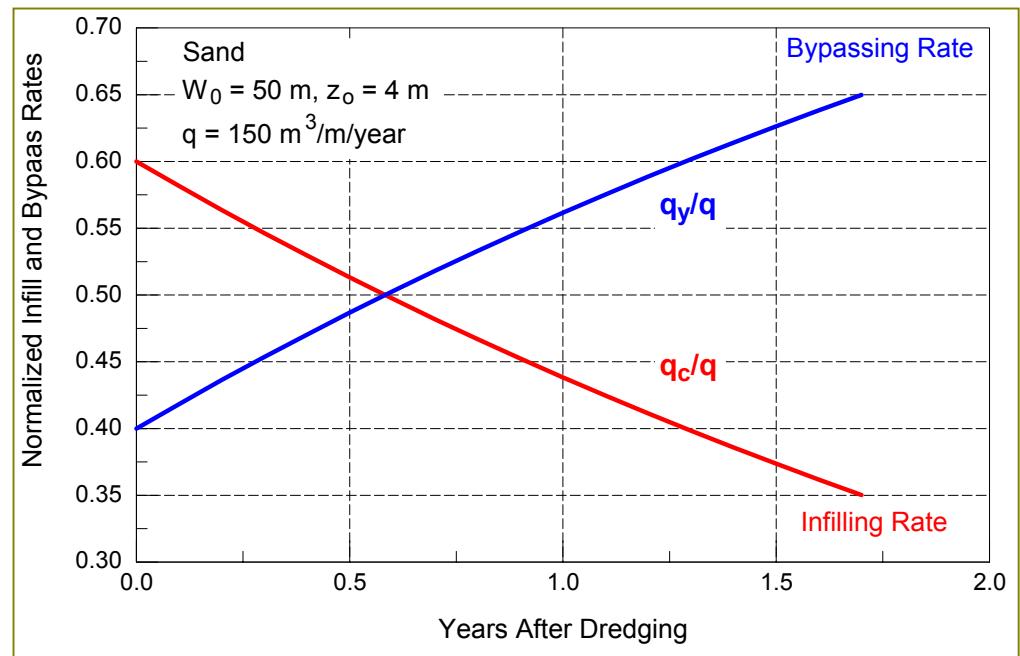
$$q_y = \left[ 1 - a_d \left( 1 - \frac{z}{z_0} \right) \right] q$$

# Example 1: Sand $a_d = 0.6$ ; $a_b = 0.1$

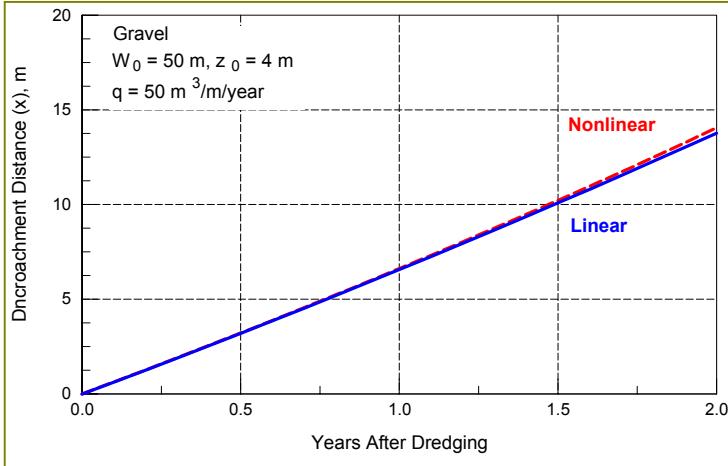
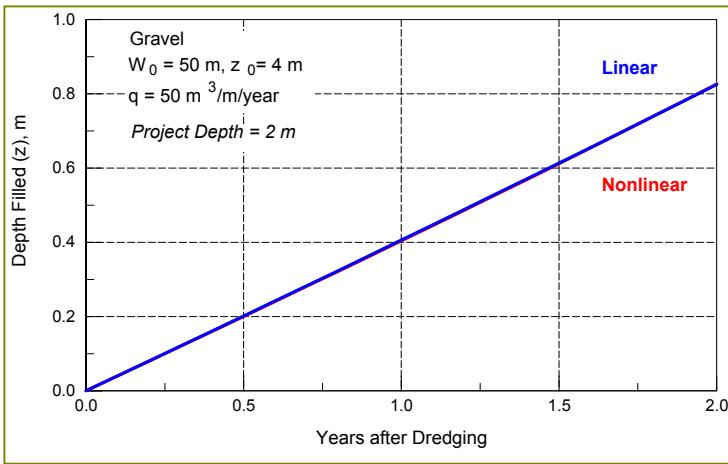


$$W_0 = 50 \text{ m}; z_0 = 4 \text{ m}; z_p = 2 \text{ m}$$

$$q = 150 \text{ m}^3/\text{m/year}$$

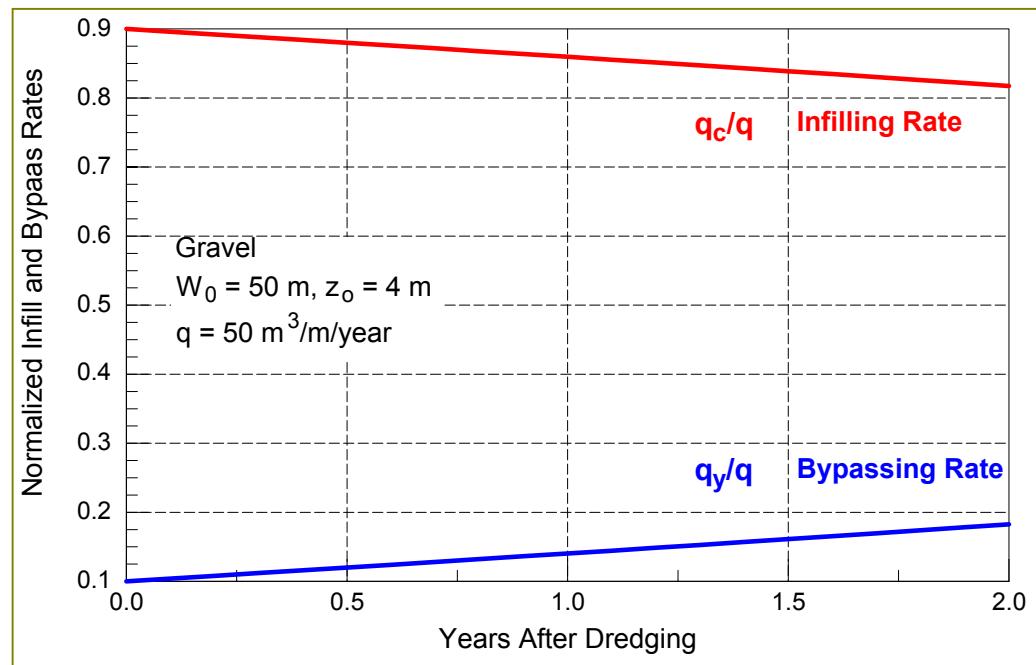


## Example 2: Gravel $a_d = 0.2$ ; $a_b = 0.7$



$$W_0 = 50 \text{ m}; z_0 = 4 \text{ m}; z_p = 2 \text{ m}$$

$$q = 50 \text{ m}^3/\text{m/year}$$



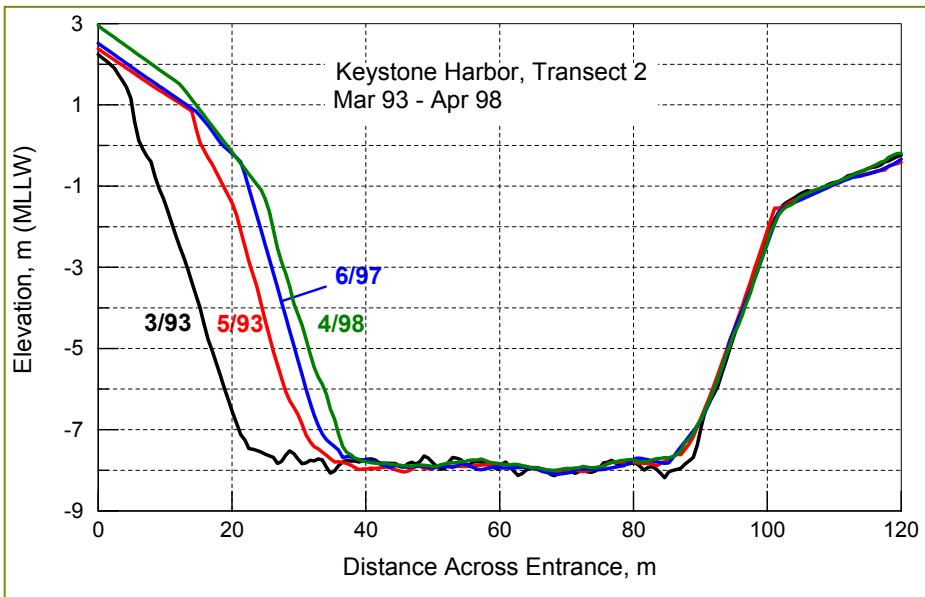
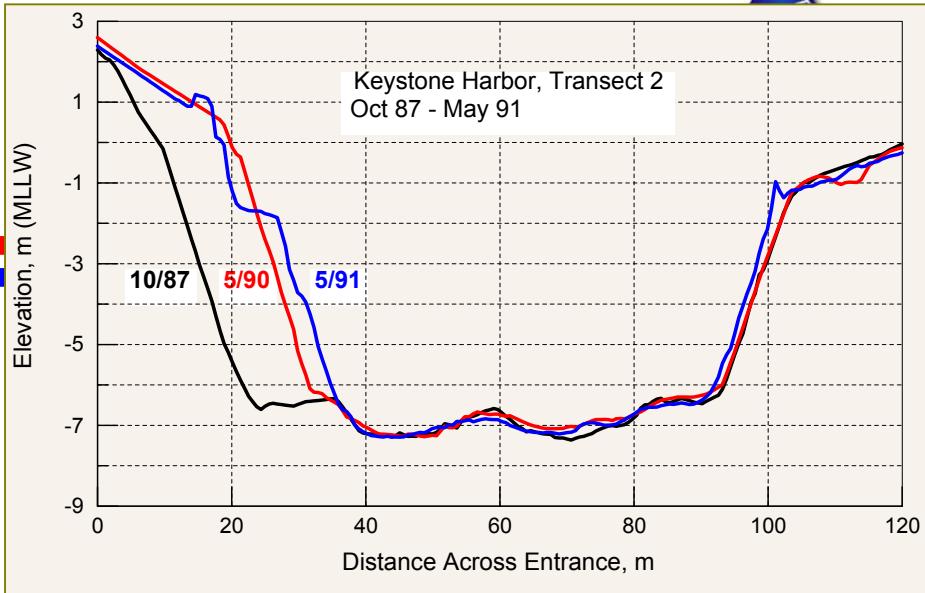
# Application – Keystone Harbor, WA



# Keystone Harbor, WA



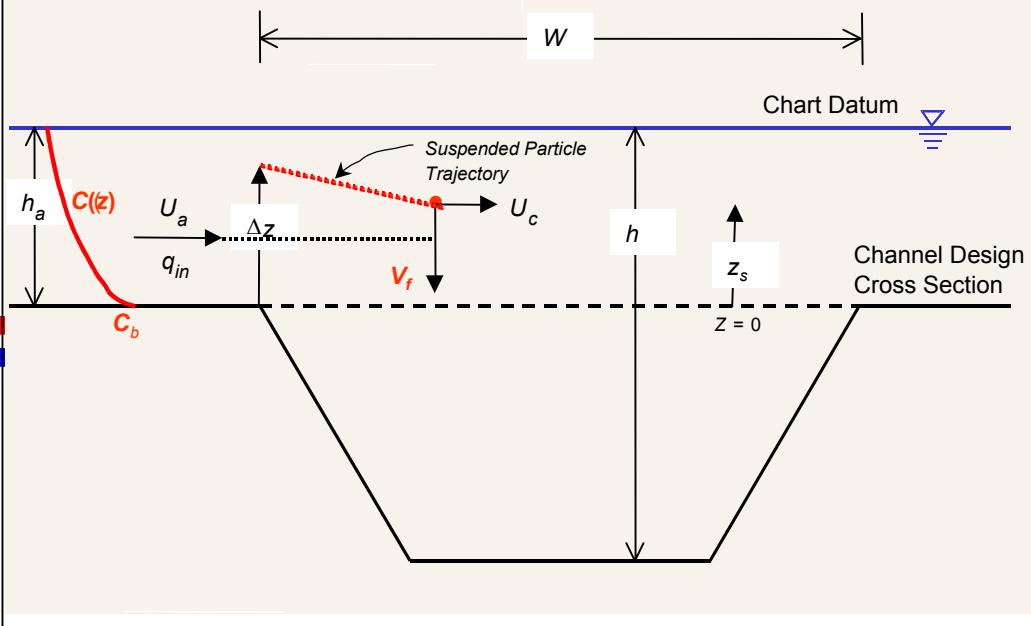
$$\begin{aligned}
 x(t) &= \frac{a_b}{z_0} qt + \frac{a_b a_d}{2W_0 z_0^2} (qt)^2 \\
 &\approx \frac{0.7}{7 \text{ m}} \left( \frac{5,000 \text{ m}^3/\text{year}}{150 \text{ m}} \right) (1 \text{ year}) \\
 &= 3.5 \text{ m}
 \end{aligned}$$



# Trapping Ratio, $p$

$$p = \frac{\text{flux deposited in channel}}{\text{total flux of suspended sediment}}$$

$$q_{in} = \int_0^{h_a} U_a C(z) dz = U_a \int_0^{h_a} C(z) dz$$



$$C(z) = C_b \exp(-\lambda z / h_a)$$

$$q_{in}(z_s) = U_a \int_0^{z_s} C_b \exp(-\lambda z / h_a) dz = \frac{U_a C_b h_a}{\lambda} [1 - \exp(-\lambda z_s / h_a)]$$

$$q_{in}(h_a) = \frac{U_a C_b h_a}{\lambda} [1 - \exp(-\lambda)] \quad U_c = \frac{h_a}{h_c} U_a \quad \Delta z = \frac{W}{U_c} V_f = \frac{h_c}{h_a} \frac{V_f}{U_a} W$$

$$p = \frac{q_{in}(\Delta z)}{q_{in}(h_a)} = \frac{1 - \exp(-\lambda \Delta z / h_a)}{1 - \exp(-\lambda)} = \frac{1 - \exp\left(-\lambda \frac{h_c W}{h_a^2} \frac{V_f}{U_a}\right)}{1 - \exp(-\lambda)}$$

# Suspended Sediment Decay Coefficient, $\lambda$

$$p = \frac{1 - \exp\left(-\lambda \frac{h_c W}{h_a^2} \frac{V_f}{U_a}\right)}{1 - \exp(-\lambda)}$$

Empirically,  $\lambda \sim 1.65 \pm 0.68$  (Kraus & Rosati 1987); but, also derive:

$$\varepsilon_s = k_d \left( \frac{D}{\rho} \right)^{1/3} h_a \quad \exp[-(V_f / \varepsilon_s) z] \Rightarrow \lambda = \frac{V_f}{k_d (D/\rho)^{1/3}}$$

$k_d \sim 0.03$  empirically

Introduce equilibrium profile concepts  $\Rightarrow \lambda = \frac{3}{4k_d} \left( \frac{V_f^2}{gh_a} \right)^{1/3}$

$$Fr = V_f / \sqrt{gh_a}$$

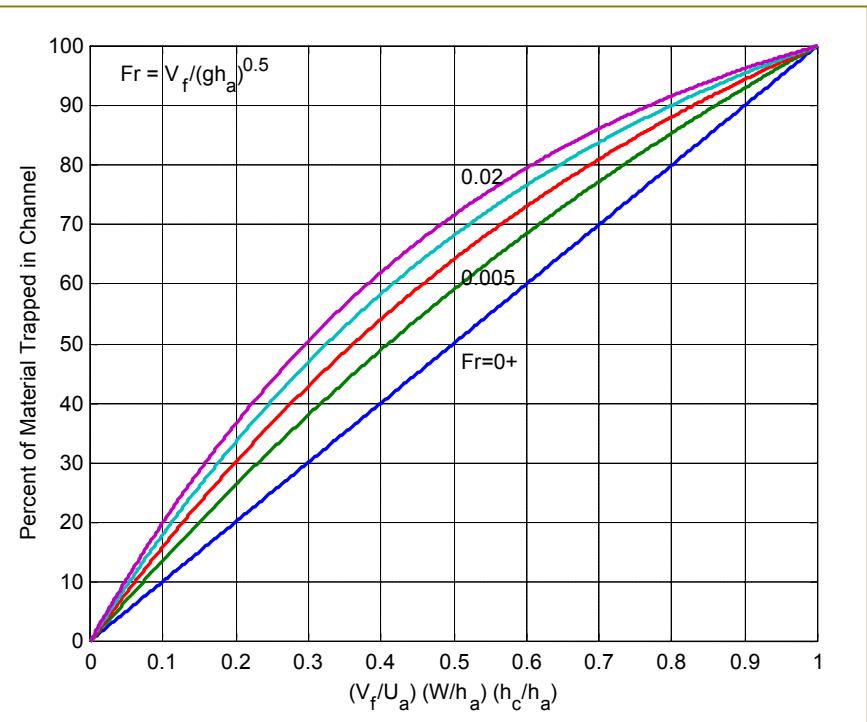
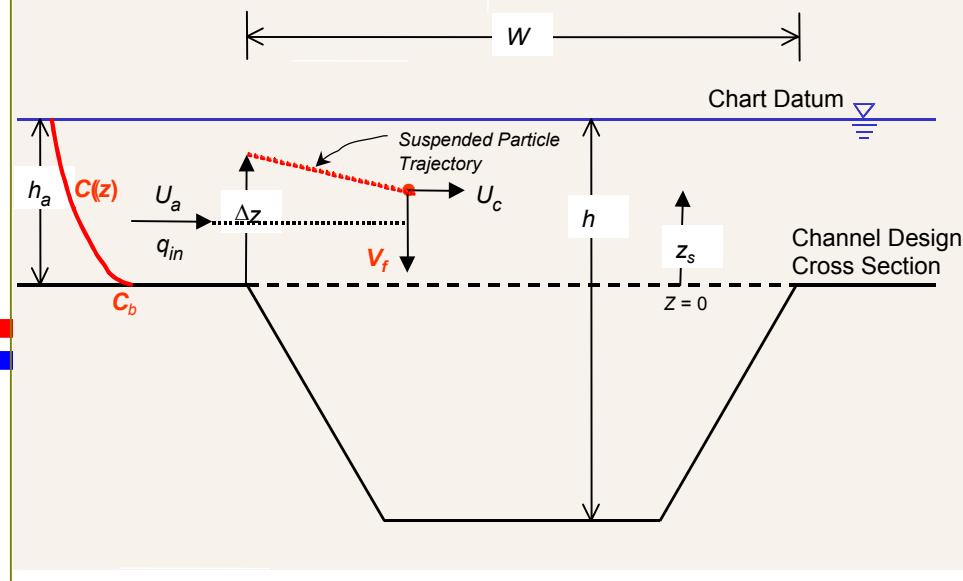
# Trapping Ratio

Practical result: trapping factor  
 $p$  = percent trapped  
 $1 - p$  = percent bypassed

$$p = \frac{1 - \exp\left(-\lambda \frac{h_c W}{h_a^2} \frac{V_f}{U_a}\right)}{1 - \exp(-\lambda)}$$

$$\lambda = \frac{3}{4k_d} \left( \frac{V_f^2}{gh_a} \right)^{1/3}$$

$$Fr = V_f / \sqrt{gh_a}$$



# Coefficient $k_d$ – analyze sediment concentration data

## Diffusion equation

$$wC = -\varepsilon_s \frac{dc}{dz}$$

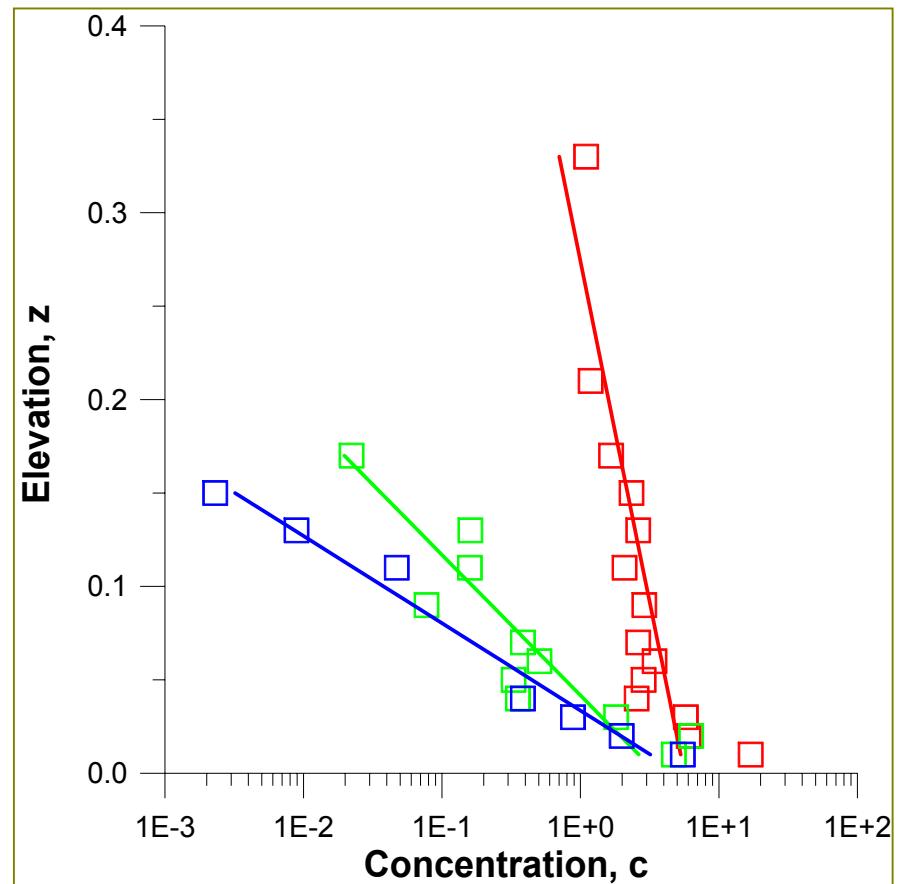
## Sediment diffusion coefficient

$$\varepsilon_s = k_d \left( \frac{D}{\rho} \right)^{1/3} h_a$$

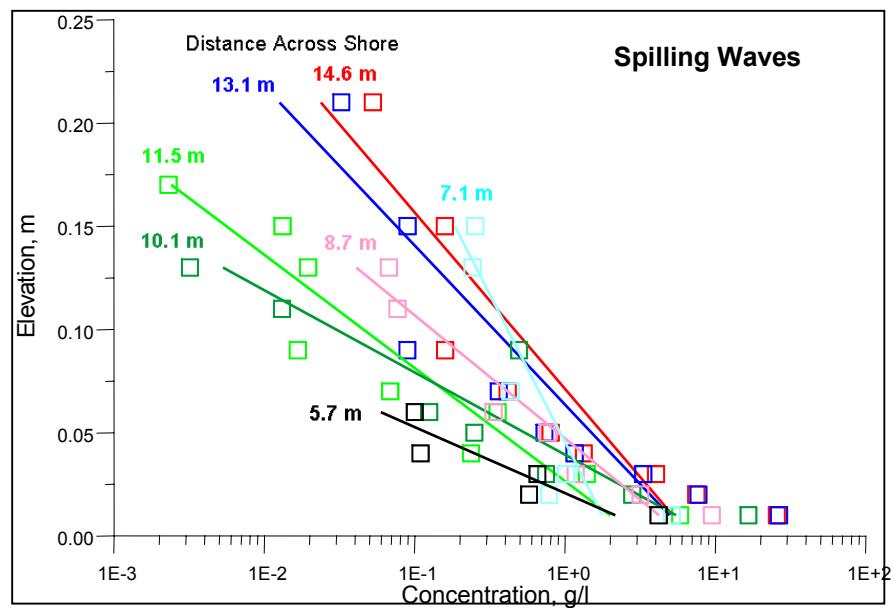
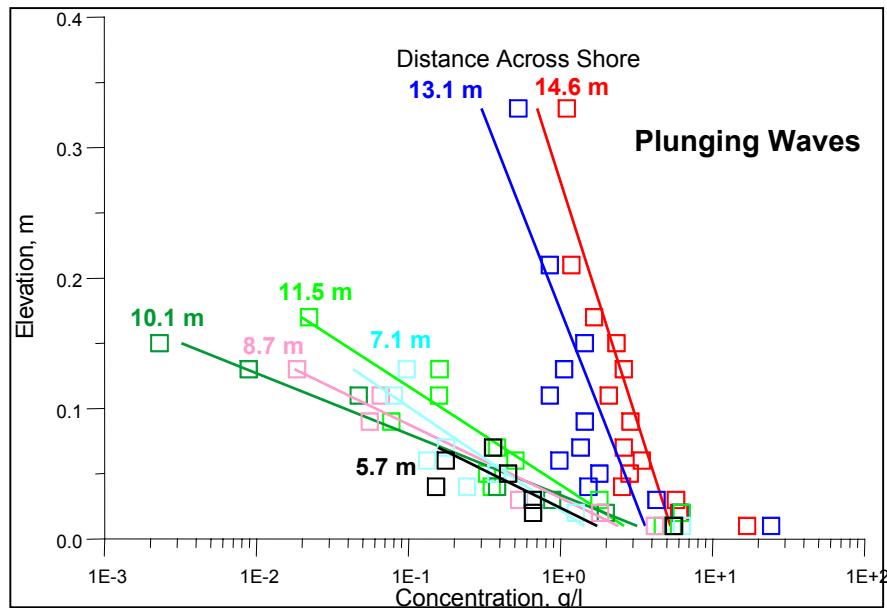
(1987 streamer trap field data,  $k_d \sim 0.03$ )

## Solution

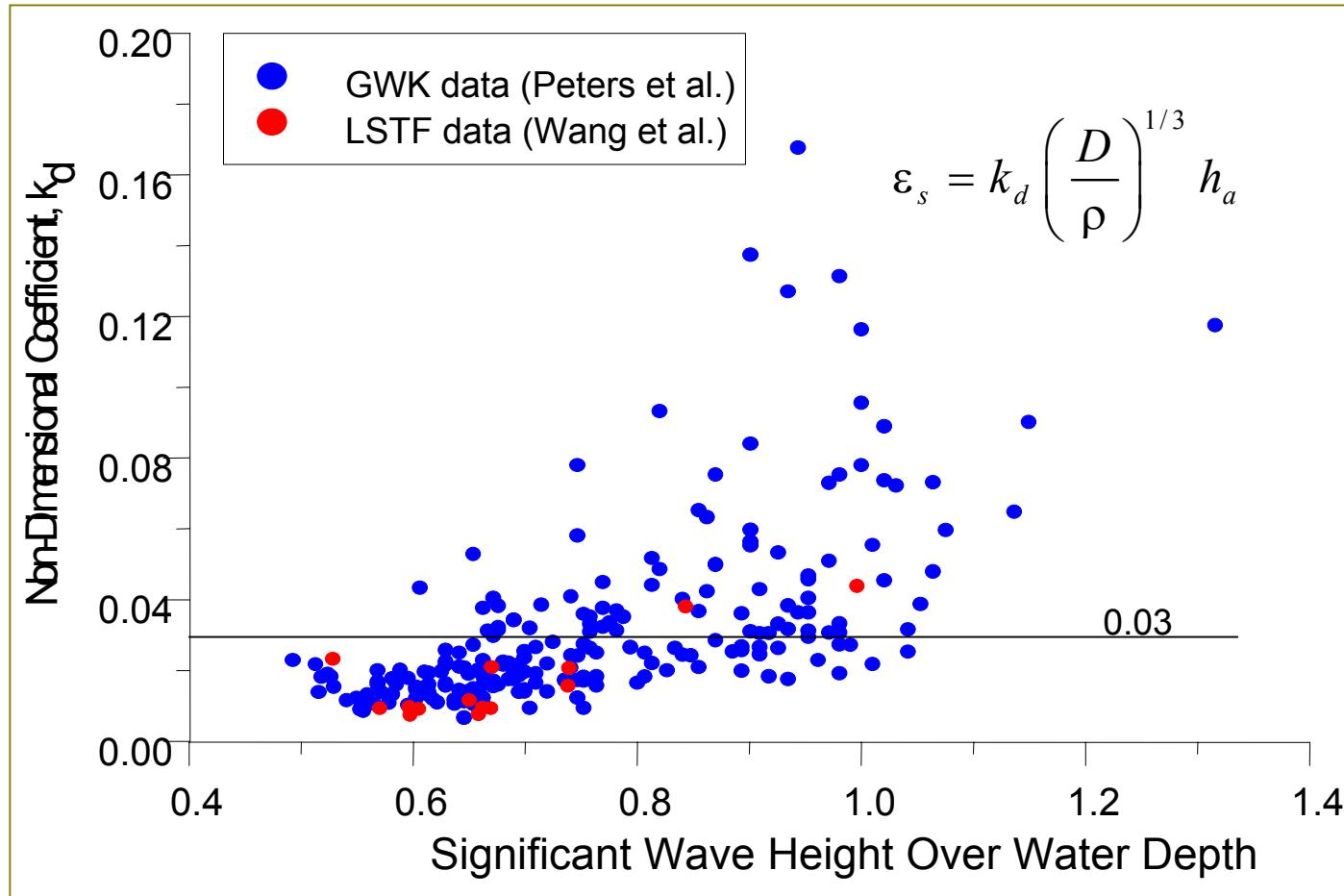
$$c = c_0 \exp \left( -\frac{w}{k_d(D/\rho)^{1/3}} \frac{z}{h_a} \right)$$



# Wang et al. (2002) mid-scale lab. data



# $k_d$ from mid-scale and large-scale lab. data



# Summary

## Navigation Channel Infilling by Cross-Channel Transport – Screening Tool –

- **Analytical model of channel infilling**
  - useful for preliminary studies
  - readily available or estimated inputs
  - isolates the acting processes
  - behavior oriented, robust
  - pedagogical tool - understand channels
  - can be extended numerically
- **Trapping ratio**
  - independent applications
  - utility for the infilling model
- **Sediment concentration profile**
  - “new” model → energy dissipation
  - validated with laboratory & field data

